Econometrica, Vol. 37, No. 4 (October, 1969)

THE RELATION BETWEEN BINARY AND MULTIPLE CHOICES: SOME COMMENTS AND FURTHER RESULTS

BY NICHOLAS GEORGESCU-ROEGEN

PROFESSOR S. K. CHAKRABARTI's note [**1**] offers me the opportunity to present some observations aimed at clarifying the relationship between the axioms introduced by Professor R. Duncan Luce in his well known monograph [**3**] and those proposed earlier by myself [**2**]. In this connection I shall prove a theorem which may be of general interest.

1

First of all, I should like to make one point clear: the impossibility proved by my Theorem 8 is not confined to linear relations. Let us observe that by repeated applications of Axiom A any binary probability $\omega(A_a, A_b)$ can be expressed as a simple sum

$$\omega(A_a, A_b) = S\omega(A_{i_1}, A_{i_2}, \ldots, A_{i_n}), \tag{1}$$

where S extends over all permutations of $(1, 2, \ldots, n)$ in which a precedes b. On the other hand, Axiom B states that

$$\omega(A_1|\mathscr{A}) = S\omega(A_1, A_{k_1}, A_{k_2}, \ldots, A_{k_{n-1}}), \tag{2}$$

where S extends over all permutations (k) of $(2, 3, \ldots, n)$. In my axiomatic basis, therefore, the probabilities of complete rankings are the *primary* variables in terms of which everything else is defined through the basic formulae, (11) and (12) of [**3**].[1] Hence, if a relation between $\omega(A_1|\mathscr{A})$ and the binary probabilities exists at all, such a relation is necessarily the result of eliminating the primary variables involved in (2) from (1) and (2). Since both (1) and (2) are linear in terms of these variables, the result of the elimination cannot be but a linear relation.[2] This spells out the reason—invoked at the beginning of my proof of Theorem 8 [**2**, p. 166]—why all we need is to prove that no linear relation exists between the variables involved.

2

There is another point that, I have found, requires emphasis: while Theorem 8 says that, within the axiomatic basis provided by Axioms A and B, there is no way of eliminating the primary variables between (1) and (2), it can say *nothing* about whether this elimination is possible in case the axiomatic basis is altered (and, of course, nothing about the result of the elimination, if feasible). As I explicitly stated, "Other assumptions than those analyzed in this paper may change our conclusions" [**2**, p. 168].

Chakrabarti's proof that Axiom A and Axiom R, i.e., Luce's Ranking Axiom [**3**, p. 69], lead to a nonlinear expression of $\omega(A|A_1, A_2, A_3)$ in terms of the binary probabilities does not refute my Theorem 8. All that it does is to show that there are answers to one of the two queries by which I concluded my paper [**2**, p. 168].[3]

[1] In this connection, it is instructive to note that in Luce's axiomatic basis the primary variables are the $\omega(A_i|\mathscr{B})$'s for all subsets $\mathscr{B}$ of $\mathscr{A}$ and $A_i \in \mathscr{B}$. The two categories of variables coincide only if $n = 2$.

[2] The fact that by hypothesis the sum of all primary variables is equal to unity need not bother us at all in connection with this algebraic elimination.

[3] The fact that such a relation may exist for axiomatic bases other than my own has been known ever since the publication of Luce's work, in which a general relation, which for $n = 3$ becomes Chakrabarti's (8), is derived from Luce's Choice Axiom [**3**, pp. 6, 16]. I may add that after the publication of my paper Luce communicated to me his relation (in the case of $n = 3$) as an example of the situation in which Theorem 8 does not hold.