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On the extrema of some statistical coefficients

Like most mathematical results, inequalities concerning the functions used in defining statistical coefficients may present only a purely mathematical interest at the time of their discovery. But this is not always so. Witness the article published in a previous issue of this Journal by Carlo Benedetti, in which the author established two very interesting inequalities concerning mathematical expressions used in measures of correlation and derived equally interesting conclusions for some topics of more immediate interest.(1) For this very reason, it seems useful to have simpler proofs of Benedetti's theorems, especially since the method used in the proofs offered in this paper is applicable to a large class of problems concerning the extrema of symmetrical, homogeneoues algebraic forms. As an additional illustration of this method, two new theorems regarding the extrema of the generalized Pearsonian coefficients β_k will be presented in the last part of the paper. These results may prove interesting to the mathematical statistician, if not also to the statistician in general.

We shall begin by Benedetti's theorems.

THEOREM I. *Let two sets of n real variables,* (x_i) *and* (y_i), *satisfy the conditions*

$$\sum_1^n x_i = \sum_1^n y_i = 0\,, \qquad \sum_1^n x_i^2 = \sum_1^n y_i^2 = 1\,, \tag{1}$$

$$x_i \leq x_{i+1}\,, \quad y_i \leq y_{i+1}\,, \qquad (i = 1, 2, \ldots n-1)\,, \tag{2}$$

(1) CARLO BENEDETTI, *Di alcune disuguaglianze collegate al campo di variazione di indici statistici*, Metron, XVIII, N. 3-4 (31-VII-1957), pp. 103-125.