NOTE ON THE ECONOMIC EQUILIBRIUM FOR NONLINEAR MODELS

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IT IS THE purpose of this note to present further results concerning the existence of economic equilibrium in the case of a model more general than that considered by either Mr. Nikaido in the note published in this issue or by this writer in *Activity Analysis of Production and Allocation.*[1]

1. The results reached by Nikaido can be obtained as an application of Theorem 2 (and its Corollaries) of Bohnenblust, Karlin, and Shapley.[2] Nikaido's note offers, however, a much more elementary proof for the particular case treated by him. Since this particular case is of special importance in the discussion of economic equilibrium any elementarization of its treatment deserves our attention.[3]

In this respect, one may point out that Nikaido's results, concerning the existence of saddle points for both $K(A, j)$ and $\phi(A, Y)$, can be proved by following step by step the procedure adopted by this writer in the paper already mentioned. The results of Theorems $I - V$ are valid, *mutatis mutandis* for either $K(A, j)$ or $\phi(A, Y)$ in case of concavity. In particular, Theorem III offers a sharpening of Nikaido's results.

Finally, it should be remarked that Nikaido's model does not constitute a real generalization of von Neumann's model since parallel to dropping the assumption of *linearity*, an important restriction of the latter, it introduces a new restriction: *limitationality*. Indeed, in Nikaido's model, to each input A there corresponds only one output $f(A)$. A reexamination of my argument in the volume on *Activity Analysis*, in the light of Nikaido's result, leads to the conclusion that the limitationality restriction can be eliminated without affecting the existence of economic equilibrium. These new results are presented in the next sections.

2. Referring to the technological horizon H as the set of all feasible *transformations* $T = (A, B)$,[4] we shall introduce the following postulates, I – III.

I. *The technological horizon forms a closed convex set in the positive orthant* R_+^{2n} *of the space of coordinates* $(a_1, a_2, \cdots, a_n, b_1, b_2, \cdots, b_n)$.

The problem of justifying this postulate is rather delicate. Some implications of (I) represent well-known interpretations of "decreasing returns." Thus:

[1] "The Aggregate Linear Production Function and its Application to von Neumann's Economic Model," *loc. cit.*, pp. 98–115.

[2] "Games with Continuous, Convex Pay-Off," *Contributions to the Theory of Games*, Vol. 1, Annals of Mathematics Studies, No. 24, pp. 181–192. (Their main Theorem 1 (p. 185) requires the additional condition that "$\inf_y \sup_\alpha \varphi_\alpha(y) < +\infty$." Indeed if $c = +\infty$, the application of Lemma 1.5 is incorrect.)

[3] *E.g.*, J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, 2nd edition, p. 154n.

[4] Georgescu-Roegen, *op. cit.*, pp. 99–102. The present paper will use the same notations, with the exception of that for a transformation, introduced above.