FURTHER CONTRIBUTION TO THE SCATTER ANALYSIS*

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1. A scatter in a p-dimensional space can be the statistical image of a single point as well as the statistical image of a $(d < p)$-dimensional variety. If the variates are the height of the parents and that of the offspring, the scatter is the image of a single point that represents the normal couple of heights. In this case, there is no true sense in speaking of a mathematical relation between the two variates. But if the scatter is formed by the observed prices and quantities under conditions of constant supply, we can logically regard the scatter as the image of the supply curve. It seems that the distinction between these different types of scatters has not so far been clearly pointed out. We are thus led to call a scatter that is the image of a d-dimensional variety, a *scatter of the dth order*.[1]

2. We shall assume that the scatter is formed by N groups of observations and that it involves $n+m$ variates $(X^1, X^2, \ldots, X^n, Y^1, Y^2, \ldots, Y^m)$, of which only the variates X are subject to sampling errors. The scatter will also be assumed to be the image of a linear variety.

3. At the beginning, we shall also assume that the deviations $\xi^i = x^i - X^i$ are distributed normally and that each ξ_k^i may be correlated to every other ξ_l^j, but that the distribution reflects the iteration of the successive deviations, *i.e.*, $b_{ij}^{kl} = E(\xi_i^k \xi_j^l) = E(\xi^k_{i+\alpha} \xi^l_{j+\alpha}) = b_{i+\alpha, j+\alpha}^{kl}$, for all values of α, $E(x)$ being the probable value of x.

If the scatter is of the $(n+m-1)$ order, *i.e.*, the theoretical variety is given by $A_k X^k + B_k Y^k + C = 0$, where $A_k X^k$ stands for $\Sigma_k A_k X_k$, it can be proved that the criterion of maximum likelihood leads to the system

$$\text{(1)} \qquad \frac{\partial H_0}{\partial A_k} = 0, \qquad \frac{\partial H_0}{\partial B_k} = 0, \qquad \frac{\partial H_0}{\partial C} = 0,$$

* Speaker was not present to read this paper.

[1] Some of the results obtained along this line of thought are to be found in the author's thesis "Le probleme de la recherche des composantes cycliques," *Journal de la Société de Statistique de Paris*, October, 1930.